**E**stimation **A**nd **C**onfidence **I**ntervals

**Background**

In quality control processes, especially when dealing with high-value items, destructive sampling is a necessary but costly method to ensure product quality. The test to determine whether an item meets the quality standards destroys the item, leading to the requirement of small sample sizes due to cost constraints.

**Scenario**

A manufacturer of print-heads for personal computers is interested in estimating the mean durability of their print-heads in terms of the number of characters printed before failure. To assess this, the manufacturer conducts a study on a small sample of print-heads due to the destructive nature of the testing process.

**Data**

A total of 15 print-heads were randomly selected and tested until failure. The durability of each print-head (in millions of characters) was recorded as follows:

1.13, 1.55, 1.43, 0.92, 1.25, 1.36, 1.32, 0.85, 1.07, 1.48, 1.20, 1.33, 1.18, 1.22, 1.29

**Assignment Tasks**

**a. Build 99% Confidence Interval Using Sample Standard Deviation**

Assuming the sample is representative of the population, construct a 99% confidence interval for the mean number of characters printed before the print-head fails using the sample standard deviation. Explain the steps you take and the rationale behind using the t-distribution for this task.

**Explanation:**

**The steps taken using the t-distribution:**

1. Sample Mean and Standard Deviation:

- The sample mean is calculated to estimate the population mean.

- The sample standard deviation is calculated to estimate the population standard deviation.

2. Standard Error:

- The standard error is calculated by dividing the sample standard deviation by the square root of the sample size.

- It represents the standard deviation of the sampling distribution of the sample mean.

3. Critical Value (t-score):

- The critical value is determined from the t-distribution based on the desired confidence level and the degrees of freedom (sample size minus one).

- The t-distribution is used because the population standard deviation is unknown, and the sample standard deviation is used as an estimate.

4. Margin of Error:

- The margin of error is calculated by multiplying the standard error by the critical value.

- It represents the maximum likely difference between the sample mean and the population mean.

5. Confidence Interval:

- The confidence interval is calculated by adding and subtracting the margin of error from the sample mean.

- It provides a range within which we are confident the population mean lies.

**Rationale for Using the t-Distribution:**

- The t-distribution is used because the population standard deviation is unknown and is estimated by the sample standard deviation.

- The t-distribution accounts for the increased uncertainty when estimating the population mean from a small sample size.

- It provides a more conservative estimate of the confidence interval compared to using the standard normal distribution (z-distribution), especially with small sample sizes where the population standard deviation is unknown.

**b. Build 99% Confidence Interval Using Known Population Standard Deviation**

If it were known that the population standard deviation is 0.2 million characters, construct a 99% confidence interval for the mean number of characters printed before failure.